

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1 A If  $f(t) = (\cos 2t + \sin t)^2$ , find  $L[f(t)]$  and hence find  $L\{e^{-2t}f(t)\}$  5B Find  $L^{-1}\left\{\frac{1}{(s-2)(s^2+4)}\right\}$  5C Obtain half-range sine series for  $f(x) = x$  in  $0 < x < 2$  5D Find  $k$  and moment generating function of the following distribution. Hence find mean and variance. 5

X	1	3	4	5
P(X)	0.4k	0.1k	0.2k	0.3k

Q2 A Find the orthogonal trajectories of the family of curves  $3x^2y - y^3 = c$  6B Find  $L\left\{t\left(\frac{\cos 2t}{e^t}\right)^2\right\}$  6C Find the Fourier series expansion for  $f(x) = 1, 0 < x < 1$   
 $=x, 1 < x < 2$  8Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ Q3 A Find  $L^{-1}\left\{\log\left(\frac{s^2+9}{\sqrt{s+8}}\right)\right\}$  6B Find the analytic function  $f(z) = u + iv$  where  $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ , using Milne-Thompson's Method. 6C Fit a parabola  $y = a + bx + cx^2$  for the following data: 8

X:	1	2	3	4	5	6	7	8	9
Y:	2	6	7	8	10	11	11	10	9

- Q4 A The first 3 moments of a distribution about 2 of the random variable X are 1, 22 and 10. Compute Mean, variance and third central moment  $\mu_3$ . 6
- B Consider the equations of regression lines  $4x-5y= -33$  and  $20x-9y - 107=0$ . Find  $\bar{x}$ ,  $\bar{y}$  and correlation coefficient r. 6

C Find  $L^{-1}\left\{\frac{(s+3)^2}{(s^2+6s+5)^2}\right\}$  8

- Q5 A Show that the Laplace transform of  $\frac{\sin 2t + \sin 3t}{t} = \pi - \tan^{-1}\left(\frac{5s}{6-s^2}\right)$ . 6
- B Find Spearman's rank correlation coefficient for the data below: 6

X:	85	74	85	50	65	78	74	60	74	90
Y:	78	91	78	58	60	72	80	55	68	70

C Obtain Fourier Series for  $f(x) = \begin{cases} \cos x, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$  8

- Q6 A If  $f(x)$  is probability density function of a continuous random variable X, find mean and variance.  $f(x) = 6(x-x^2)$ ,  $0 < x < 1$  6
- B Check if there exists an analytic function whose real part is  $u = e^x(x \cos 2y - y \sin 2y)$ . If so, find  $f(z)$ . Justify your answer. 6

C Evaluate the following integral by using Laplace transforms 8

$$\int_0^{\infty} e^{-t} \left[ \int_0^t u^2 \sinh u \cosh u \, du \right] dt$$

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